

19-06-2018

Exercise 4

1	2	3 G
4		
5		
6	7	
8	9	10

The minimum horizon required to reach the Goal from any initial condition is 8. (From square #10, it takes 8 steps to reach square #3)

We define the stage cost through the following table

position	action	U	D	L	R	S
1		/	1	/	1	/
2		/	/	1	1	/
3		/	/	1	/	0
4		1	1	/	/	/
5		1	1	/	/	/
6		1	1	/	1	/
7		/	1	1	/	/
8		1	/	/	1	/
9		1	/	1	1	/
10		/	/	1	/	/

$\leftarrow g_i(x,u), i = 0, \dots, 7$

$$g_8(x) = \begin{cases} \infty & x_8 = 1 \\ \infty & x_8 = 2 \\ 0 & x_8 = 3 \\ \infty & x_8 = 4 \\ \infty & x_8 = 5 \\ \infty & x_8 = 6 \\ \infty & x_8 = 7 \\ \infty & x_8 = 8 \\ \infty & x_8 = 9 \\ \infty & x_8 = 10 \end{cases}$$

Note: we can also choose $g_8(2) = 1$ considering that is 1 step from the goal.

$$J_7(x_7) = \begin{cases} \min\left\{ \underset{D}{1+\infty}, \underset{R}{1+\infty} \right\} = \infty & x_7 = 1 \\ \min\left\{ \underset{L}{1+\infty}, \underset{B}{1+0} \right\} = 1 & x_7 = 2 \\ \min\left\{ \underset{L}{1+\infty}, \underset{S}{0+0} \right\} = 0 & x_7 = 3 \\ \min\left\{ \underset{U}{1+\infty}, \underset{D}{1+\infty} \right\} = \infty & x_7 = 4 \\ \min\left\{ \underset{U}{1+\infty}, \underset{D}{1+\infty} \right\} = \infty & x_7 = 5 \\ \min\left\{ \underset{U}{1+\infty}, \underset{D}{1+\infty}, \underset{R}{1+\infty} \right\} = \infty & x_7 = 6 \\ \min\left\{ \underset{D}{1+\infty}, \underset{L}{1+\infty} \right\} = \infty & x_7 = 7 \\ \min\left\{ \underset{U}{1+\infty}, \underset{R}{1+\infty} \right\} = \infty & x_7 = 8 \\ \min\left\{ \underset{U}{1+\infty}, \underset{L}{1+\infty}, \underset{R}{1+\infty} \right\} = \infty & x_7 = 9 \\ \min\left\{ \underset{L}{1+\infty} \right\} = \infty & x_7 = 10 \end{cases} \rightarrow J_7(x_7) = \begin{cases} \infty & x_7 = 1 \\ 1 & x_7 = 2 \\ 0 & x_7 = 3 \\ \infty & x_7 = 4 \\ \infty & x_7 = 5 \\ \infty & x_7 = 6 \\ \infty & x_7 = 7 \\ \infty & x_7 = 8 \\ \infty & x_7 = 9 \\ \infty & x_7 = 10 \end{cases}$$

$$\mu_7(x_7) = \begin{cases} D/R & x_7 = 1 \\ R & x_7 = 2 \\ S & x_7 = 3 \\ D/U & x_7 = 4 \\ D/U & x_7 = 5 \\ R/D/U & x_7 = 6 \\ L/D & x_7 = 7 \\ R/U & x_7 = 8 \\ R/L/U & x_7 = 9 \\ L & x_7 = 10 \end{cases}$$

The / indicates that the different actions would lead to the same cost. Any of them is a valid solution.